Logarithms to Base 2

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Graphical representation of the growth of a bacterial culture is usually best achieved by using a logarithmic scale for whatever measurement of the density of the culture is made. Moreover, as Monod (1949) has pointed out, both diagrams and associated calculations are more readily interpreted if logarithms to base 2 are used instead of common logarithms (i.e. base 10): an increase of 1 unit on the logarithmic scale then corresponds to the equivalent of one division by each of the cells present at the beginning of the interval.

Rather surprisingly, none of the better-known collections of mathematical tables nor any of the usual sources for miscellaneous tables appears to contain logarithms to base 2. Of course, for any particular number, multiplication of its common logarithm by 3.32193 ($=\log_2 10$) gives the logarithm to base 2, but performance of this multiplication for many series of data is tedious and time consuming. As the result of a suggestion from Dr W. E. van Heyningen, the following table has therefore been computed.† Only 3 places of decimals are shown, since graphical work will certainly not permit greater accuracy and even for arithmetical analysis observational data rarely justify the use of more digits; the table is designed for simple rapid use by microbiologists, not for serious mathematical purposes.

The table shows the logarithm of every integer from 100 to 999. Thus, by direct reading,

$$\log_2 673 = 9.394.$$

If the highest and lowest densities of a series do not differ by more than a factor of 10, there may be no necessity to construct any logarithms outside the range of the table. For example, if the densities range from 1.7 to 8.9, expression of density in terms of a volume unit 100 times that of the standard will give values ranging from 170 to 890; all logarithms can then be read directly from the table and graphed as they stand.

If the densities extend over a wider range, logarithms of numbers not in the table will be needed. These are obtained by the usual rule that the logarithm of a product or quotient is the sum or difference of the logarithms of the two parts. Thus

$$\begin{aligned}\log_2 2692 &= \log_2 4 + \log_2 673 \\ &= 2.000 + 9.394 = 11.394, \\ \log_2 0.075 &= \log_2 300 - \log_2 400 - \log_2 10 \\ &= 8.229 - 8.644 - 3.322 = -3.737.\end{aligned}$$

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† The table was obtained by multiplication of appropriate entries in the *U.S. National Bureau of Standards Table of Natural Logarithms* by $\log_2 e$ and independently checked by multiplication of entries in *Chamber's Mathematical Tables* (Seven Figure Logarithms) by $\log_2 10$.

When suitable factors are not obvious, powers of 10 can be used, remembering that $\log_2 10 = 3.3219$, $\log_2 100 = 6.6439$, $\log_2 1000 = 9.9658$; logarithms of powers of 10 up to 10^{10} are appended to the table. Hence

$$\begin{aligned}\log_2 0.875 &= \log_2 875 - \log_2 1000 \\ &= 9.773 - 9.966 = -0.193.\end{aligned}$$

(Since $0.875 = 7/8$, the same value can be obtained as $\log_2 700 - \log_2 800$.) The measurements will rarely be accurate to more than 3 significant digits, but, if required, logarithms of quantities intermediate between tabular values can be derived by the common-sense application of 'linear interpolation'; thus $\log_2 2692$ can alternatively be calculated as at 2/10 of the distance between

$$\log_2 2690 = 8.071 + 3.322 = 11.393,$$

and

$$\log_2 2700 = 8.077 + 3.322 = 11.399,$$

and is 11.394 as before.

REFERENCE

MONOD, J. (1949). The growth of bacterial cultures. *Annu. Rev. Microbiol.* 3, 371.

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	0	1	2	3	4	5	6	7	8	9
100	6·644	6·658	6·672	6·687	6·700	6·714	6·728	6·741	6·755	6·768
110	·781	·794	·807	·820	·833	·845	·858	·870	·883	·895
120	·907	·919	·931	·943	·954	·966	·977	·989	7·000	7·011
130	7·022	7·033	7·044	7·055	7·066	7·077	7·087	7·098	·109	·119
140	·129	·140	·150	·160	·170	·180	·190	·200	·209	·219
150	7·229	7·238	7·248	7·257	7·267	7·276	7·285	7·295	7·304	7·313
160	·322	·331	·340	·349	·358	·366	·375	·384	·392	·401
170	·409	·418	·426	·435	·443	·451	·459	·468	·476	·484
180	·492	·500	·508	·516	·524	·531	·539	·547	·555	·562
190	·570	·577	·585	·592	·600	·607	·615	·622	·629	·637
200	7·644	7·651	7·658	7·665	7·672	7·679	7·687	7·693	7·700	7·707
210	·714	·721	·728	·735	·741	·748	·755	·762	·768	·775
220	·781	·788	·794	·801	·807	·814	·820	·827	·833	·839
230	·845	·852	·858	·864	·870	·877	·883	·889	·895	·901
240	·907	·913	·919	·925	·931	·937	·943	·948	·954	·960
250	7·966	7·972	7·977	7·983	7·989	7·994	8·000	8·006	8·011	8·017
260	8·022	8·028	8·033	8·039	8·044	8·050	·055	·061	·066	·071
270	·077	·082	·087	·093	·098	·103	·109	·114	·119	·124
280	·129	·134	·140	·145	·150	·155	·160	·165	·170	·175
290	·180	·185	·190	·195	·200	·205	·209	·214	·219	·224
300	8·229	8·234	8·238	8·243	8·248	8·253	8·257	8·262	8·267	8·271
310	·276	·281	·285	·290	·295	·299	·304	·308	·313	·317
320	·322	·326	·331	·335	·340	·344	·349	·353	·358	·362
330	·366	·371	·375	·379	·384	·388	·392	·397	·401	·405
340	·409	·414	·418	·422	·426	·430	·435	·439	·443	·447
350	8·451	8·455	8·459	8·464	8·468	8·472	8·476	8·480	8·484	8·488
360	·492	·496	·500	·504	·508	·512	·516	·520	·524	·527
370	·531	·535	·539	·543	·547	·551	·555	·558	·562	·566
380	·570	·574	·577	·581	·585	·589	·592	·596	·600	·604
390	·607	·611	·615	·618	·622	·626	·629	·633	·637	·640
400	8·644	8·647	8·651	8·655	8·658	8·662	8·665	8·669	8·672	8·676
410	·679	·683	·687	·690	·693	·697	·700	·704	·707	·711
420	·714	·718	·721	·725	·728	·731	·735	·738	·741	·745
430	·748	·752	·755	·758	·762	·765	·768	·771	·775	·778
440	·781	·785	·788	·791	·794	·798	·801	·804	·807	·811
450	8·814	8·817	8·820	8·823	8·827	8·830	8·833	8·836	8·839	8·842
460	·845	·849	·852	·855	·858	·861	·864	·867	·870	·873
470	·877	·880	·883	·886	·889	·892	·895	·898	·901	·904
480	·907	·910	·913	·916	·919	·922	·925	·928	·931	·934
490	·937	·940	·943	·945	·948	·951	·954	·957	·960	·963
500	8·966	8·969	8·972	8·974	8·977	8·980	8·983	8·986	8·989	8·992
510	·994	·997	9·000	9·003	9·006	9·008	9·011	9·014	9·017	9·020
520	9·022	9·025	·028	·031	·033	·036	·039	·042	·044	·047
530	·050	·053	·055	·058	·061	·063	·066	·069	·071	·074
540	·077	·079	·082	·085	·087	·090	·093	·085	·098	·101

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	0	1	2	3	4	5	6	7	8	9
550	9·103	9·106	9·109	9·111	9·114	9·116	9·119	9·122	9·124	9·127
560	·129	·132	·134	·137	·140	·142	·145	·147	·150	·152
570	·155	·157	·160	·162	·165	·167	·170	·172	·175	·177
580	·180	·182	·185	·187	·190	·192	·195	·197	·200	·202
590	·205	·207	·210	·212	·214	·217	·219	·222	·224	·226
600	9·229	9·231	9·234	9·236	9·238	9·241	9·243	9·246	9·248	9·250
610	·253	·255	·257	·260	·262	·264	·267	·269	·271	·274
620	·276	·278	·281	·283	·285	·288	·290	·292	·295	·297
630	·299	·301	·304	·306	·308	·311	·313	·315	·317	·320
640	·322	·324	·326	·329	·331	·333	·335	·338	·340	·342
650	9·344	9·347	9·349	9·351	9·353	9·355	9·358	9·360	9·362	9·364
660	·366	·369	·371	·373	·375	·377	·379	·382	·384	·386
670	·388	·390	·392	·394	·397	·399	·401	·403	·405	·407
680	·409	·412	·414	·416	·418	·420	·422	·424	·426	·428
690	·430	·433	·435	·437	·439	·441	·443	·445	·447	·449
700	9·451	9·453	9·455	9·457	9·459	9·461	9·464	9·466	9·468	9·470
710	·472	·474	·476	·478	·480	·482	·484	·486	·488	·490
720	·492	·494	·496	·498	·500	·502	·504	·506	·508	·510
730	·512	·514	·516	·518	·520	·522	·524	·526	·527	·529
740	·531	·533	·535	·537	·539	·541	·543	·545	·547	·549
750	9·551	9·553	9·555	9·557	9·558	9·560	9·562	9·564	9·566	9·568
760	·570	·572	·574	·576	·577	·579	·581	·583	·585	·587
770	·589	·591	·592	·594	·596	·598	·600	·602	·604	·605
780	·607	·609	·611	·613	·615	·617	·618	·620	·622	·624
790	·626	·628	·629	·631	·633	·635	·637	·638	·640	·642
800	9·644	9·646	9·647	9·649	9·651	9·653	9·655	9·656	9·658	9·660
810	·662	·664	·665	·667	·669	·671	·672	·674	·676	·678
820	·679	·681	·683	·685	·687	·688	·690	·692	·693	·695
830	·697	·699	·700	·702	·704	·706	·707	·709	·711	·713
840	·714	·716	·718	·719	·721	·723	·725	·726	·728	·730
850	9·731	9·733	9·735	9·736	9·738	9·740	9·741	9·743	9·745	9·747
860	·748	·750	·752	·753	·755	·757	·758	·760	·762	·763
870	·765	·767	·768	·770	·771	·773	·775	·776	·778	·780
880	·781	·783	·785	·786	·788	·790	·791	·793	·794	·796
890	·798	·799	·801	·803	·804	·806	·807	·809	·811	·812
900	9·814	9·815	9·817	9·819	9·820	9·822	9·823	9·825	9·827	9·828
910	·830	·831	·833	·834	·836	·838	·839	·841	·842	·844
920	·845	·847	·849	·850	·852	·853	·855	·856	·858	·860
930	·861	·863	·864	·866	·867	·869	·870	·872	·873	·875
940	·877	·878	·880	·881	·883	·884	·886	·887	·889	·890
950	9·892	9·893	9·895	9·896	9·898	9·899	9·901	9·902	9·904	9·905
960	·907	·908	·910	·911	·913	·914	·916	·917	·919	·920
970	·922	·923	·925	·926	·928	·929	·931	·932	·934	·935
980	·937	·938	·940	·941	·943	·944	·945	·947	·948	·950
990	·951	·953	·954	·956	·957	·959	·960	·961	·963	·964
10	10 ¹	10 ²	10 ³	10 ⁴	10 ⁵	10 ⁶	10 ⁷	10 ⁸	10 ⁹	10 ¹⁰
3·322	6·644	9·966	13·288	16·610	19·932	23·253	26·575	29·897	33·219	